Delta neutral G function of Meijer and H function of Fox in the neighbourhood of finite branch point

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Meijer's G function $G^{m,n}_{p,q}(z)$ is called delta neutral if $p = q = m + n$. It is a particular case delta neutral Fox's $H^{m,n}_{p,q}$ function, defined by the conditions that $\Delta = \sum_{i=1}^{q} B_i - \sum_{i=1}^{p} A_i = 0$ and $\sum_{i=1}^{m} B_i = \sum_{i=n+1}^{p} A_i$, where $A_i, B_i > 0$ are scaling factors in the gamma functions in the integrand of the H function. The delta neutral H function is comprised of two analytic functions, one defined inside the disk $|z| < \rho$ and the other outside this disk, where $\rho = \prod_{i=1}^{q} B_i^{B_i} \prod_{j=1}^{p} A_j^{-A_j}$. These functions are not analytic continuations of each other. The point $z = \rho$ (in case of G function $\rho = 1$) is, generally speaking, a branch point of both "inner" and "outer" functions. In the talk we discuss the basic case of the delta neutral $H^{p,0}_{p,q}(z)$. The behavior of its particular case $G^{p,0}_{p,p}(z)$ in the neighborhood of $z = 1$ was studied by Nørlund. We generalize some of his results to the H function case. In particular, we present an expansion in the neighborhood of the branch point $z = \rho$, give a new formula for the Mellin transform for some exceptional parameters, derive a new integral equation satisfied by this function and propose a conjecture regarding its zeros. At the core of some of our proofs lies a (presumably new) expansion in the inverse factorial series for the integrand of the H function. Furthermore, we discuss positivity of the delta neutral H function and present some new facts for Meijer's G function.

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